**Nipissing University Senior Mathematics Competition**

Problem Set 3 – Complex Numbers – May 7, 2010

1. Determine *x* and *y*, real numbers such that the equation below should be satisfied:

$$\frac{x+1}{1+i}+\frac{y-1}{1-i}=\frac{1-7i}{2}$$

1. Calculate and/or simplify. Give your answers in the form $x+yi$
a) $\frac{-3+i}{5-i}-\frac{4-5i}{5+i}=$

b) $\left(\frac{1+i\sqrt{3}}{3}\right)^{3}-\left(\frac{1-i\sqrt{3}}{3}\right)^{3}=$

c) $i^{24m+3}+i^{17}+i^{8m+2}+i^{16}=$

1. Interpret geometrically and show on the complex plane the following relations:
2. $\left|3+4i\right|=5$
3. $\left(3-7i\right)+\left(1+5i\right)=4-2i$



1. Determine *z* such that all complex numbers *z*  and $\frac{1}{z}$ must have the same modulus and $z+\overbar{z}=z\overbar{z}$.
2. Determine all the ordered pairs $(x,y)$ in the Cartesian plane such that:

$$\left|\sqrt{3x+2y}+i\sqrt{2x+3y}\right|=\sqrt{5}$$

1. Solve the following equations
2. $1+\sqrt{x+1}-2\sqrt[4]{x+1}=0$
3. $\sqrt[4]{17-x}+\sqrt[4]{x}=3$
4. Let $P\left(z\right)=z^{2}+a\left|z\right|+b, z\in C$**.** Determine *a* and *b* such that $P\left(1\right)=P\left(2\right)=2$, and find all values of z for which P(z)=0.